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The Influence of Uniform Magnetic Field in the Estimation of the Critical Permeability in a 2D- Bio-Porpus Convection

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Abstract—Bioconvection is a new area of fluid mechanics and it describes the phenomenon of spontaneous pattern formation in suspensions of micro-organisms like Bacillus subtilis and algae. The term "bioconvection" refered to macroscopic convection induced in water by the collective motion of a large number of self propelled motile micro-organisms. The direction of swimming micro-organisms is dictated by the different tactic nature of the micro-organisms. They may be gyrotactic, phototactic, chemotactic, etc.. The present study deals with the linear stability analysis of a suspension of gyrotactic micro-organisms in a horizontal sparsely packed porous fluid layer of finite depth subject to adverse temperature gradient. This problem is relevant to certain species of thermophilic micro-organisms that live in hot environment. Finally it is concluded that the permeability as well as the Uniform magnetic field have strong opposing influences on the onset of bio-convection.Thus, by controlling these parameters it is possible to enhance or suppress bio-convection.

Keywords-component; Bio-porous convection, eccentricity,critical permeability microorganism, critical Rayleigh number and critical wave number, uniform magnetic field

1. Introduction

Bioconvection is the name given to pattern formation in suspensions of microorganisms, such as bacteria and algae, due to up-swimming of the micro-organisms (Pedley & Kessler 1992). Bioconvection has been observed in several bacterial species, including aerobic, anaerobic, and magnetotactic organisms, as well as in algal and protozoan cultures (Kessler and Hill (1997)). All have in common the sudden appearance of a pattern when viewed from above. In all cases the microorganisms are denser than water and on average they swim upwards (although the reasons for up-swimming may be different for different species). The algae (e.g. Chlamydomonas) are approximately 5% denser than water, whereas the bacteria are nearly 10% denser than water. Microorganisms respond to certain stimuli by tending to swim in particular directions. These responses are called taxes, examples being gravitaxis, phototaxis, chemotaxis and gyrotaxis. Gyrotaxis is swimming directed by the balance between the torque due to gravity acting on a bottom-heavy cell and the torque due to viscous forces arising from local shear flows. This chapter is concerned with the 2D stability analysis of bioconvection in a suspension of motile gyrotactic microoganisms in an electrically conducting fluid saturated porous medium subject to magnetic field . The method employed is perturbation technique and the results are obtained by using the computational tools viz. **Maple and Mathematica**..

Nomenclature : C_a : The acceleration coefficient. D : Tthe diffusivity of the microorganisms (this assumes that all random motions of the microorganisms can be approximated by a diffusive process). g : The gravitational acceleration. n : The number density of motile microorganisms. n_o : The number density of microorganisms in the basic state. p : The excess pressure(above hydrostatic) $\hat{p}$: The unit vector indicating the direction of swimming of microorganisms . t : The time; u, v and w are the x-,y and z-velocity components respectively. $\vec{v}$: The velocity vector: (u,v,w) . $W_c\hat{p}$: The vector of average swimming velocity of microorganisms relative to the fluid (W_c is assumed to be constant). x y and z : The Cartesian coordinates (y is vertical coordinate). θ : The average volume of microorganisms. $\Delta\rho$: The density difference $\rho_{cell}-\rho_0$. μ : The dynamic viscosity. assumed to be approximately the same as that of water. ρ_o : The density of water. μ^* = Magnetic parameter

2 Mathematical formulation and analysis

The governing equations are : the momentum equation in the component form;

$$C_a\rho_o\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}-\frac{\mu w}{K}-n\theta\Delta\rho g-\mu^*\frac{\partial\vec{H}}{\partial z}\quad ..(1)$$ *where* $\vec{H}=(0,0,H)$ where H is the uniform magnetic field

$C_a\rho_o \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \frac{\mu u}{K}$ ..(2) ,The continuity equation: $\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$ ..(3) the conservation of cells $\frac{\partial n}{\partial t} = -div(n\vec{u} + nw_c\vec{p} - D\Delta n)$ ..(4) magnetic induction equation is $\frac{\partial \vec{H}}{\partial t} = \frac{\partial w}{\partial z} + \vartheta_m \nabla^2 \vec{H}$ ..(5)

3 **Pressure elimination :** consider $\frac{\partial(2)}{\partial x} - \frac{\partial(1)}{\partial x}$

$$\Rightarrow C_a\rho_o \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = -\frac{\mu}{K}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \theta\Delta\rho g \frac{\partial n}{\partial x} + \mu^* \frac{\partial^2 H}{\partial x \partial z} \quad ..(6)$$

4 Stability analysis In order to obtain the stability criterion the perturbations of cell concentration, fluid velocity components and the unit vector $\vec{p}$ that indicates the direction of bacterial swimming are introduced as follows $n(t,x,z) = n_o + \varepsilon n^1(t,x,z)$, ..(7) $u(t,x,z) = \varepsilon u^1(t,x,z)$, ..(8) $w(t,x,z) = \varepsilon w^1(t,x,z)$, ..(9) $\vec{p}(t,x,z) = \hat{k} + \varepsilon\vec{p}^1(t,x,z)$ ..(10) $\vec{H}(t,x,z) = H_o + \varepsilon H^1(t,x,z)$, ..(11) where $\hat{k}$ is the vector in the vertical direction. a prime denoted a perturbation quantity, $\vec{H}^1$ is the perturbation in the magnetic field and ε is the small perturbation amplitude After eliminating the pressure the perturbations (7-11) are substituted in to the governing equations which result in the following set of linearized equations: From equation (5).

$C_a\rho_o \frac{\partial}{\partial t}\left(\frac{\partial u^1}{\partial z} - \frac{\partial w^1}{\partial x}\right) = -\frac{\mu}{K}\left(\frac{\partial u^1}{\partial z} - \frac{\partial w^1}{\partial x}\right) + \theta\Delta\rho g \frac{\partial n^1}{\partial x} + \mu^* \frac{\partial^2 H^1}{\partial x \partial z}$. .(12) and $\frac{\partial u^1}{\partial x} + \frac{\partial v^1}{\partial y} = 0$ ..(`13)

From(4) $\frac{\partial n^1}{\partial t} = -div\left[n_o\left(\vec{u}^1 + w_c\vec{p}^1\right) + w_c n^1\hat{k} - D\nabla n^1\right]$ ..(14) from $\Rightarrow \frac{\partial}{\partial t}\left(H^1\right) = \frac{\partial w^1}{\partial z} + \vartheta_m \frac{\partial^2 H^1}{\partial z^2}$ ..(15)

where $\vec{u}^1$ is the vector composed of perturbations of the corresponding velocity components, (u', w^1). Pedly et al. [1988] analyzed the impact of the gyrotaxes on the direction of swimming of microorganisms. Then obtained an equation that relates the perturbation of the swimming direction. $\vec{p}^1$ with perturbation of velocity components. For a two dimensional problem, the results of pedly et al. [1988] can be presented as

$\vec{p}^1 = (-B\xi, 0)$ ..(16)

where B is the time scale for the reorientation of microorganisms by the gravitational torque against viscous resistance. In Pedley and Kessler [1987] this parameter is called the "gyrotactic orientation parameter", It can be expressed as $B = \frac{\alpha_\perp \mu}{2h\rho_o g}$ ...(17) where $\alpha_\perp$ is the dimensionless constant relating viscous torque to the relative angular velocity of the cell and h was the displacement of centre of mass of the cell from the centre of buoyancy. The components ξ of vector $\hat{p}^1$ in eqn.(16) is connected to perturbations of velocity component by the following equation: $\xi = (1+\alpha_o)\frac{\partial u^1}{\partial z} - (1-\alpha_o)\frac{\partial w^1}{\partial x}$ ..(18) *α_o is the cell eccentricity which is given by the following equation.* $\alpha_o = \frac{a^2 - b^2}{a^2 + b^2}$ *..(19)*

*where a and b are the semi-major and semi-minor axes of the spheroidal cell respectively.*By substituting eqn,(16) into eqn.(14) and accounting the eqn. (18), the following equation for n' is obtained.

$$\frac{\partial n^1}{\partial t} + w_c \frac{\partial n^1}{\partial z} + w_c B n_o \left(\frac{\partial \xi}{\partial x}\right) = D\nabla^2 n^1 \quad ..(20)$$

In order to study the stability of the system, the perturbation quantities are introduced in terms of individual Fourier modes: n^1 (t,x ,z)=N exp[$\sigma t + i(lx+mz)$] .. (21)

w^1(t,x,z)=Uexp[$\sigma t + i(lx+mz)$] ..(22)

H^1(t,x,z)=Qexp[$\sigma t + i(lx+mz)$] ..(23.) Again by substituting eqn.(22) into continuity equation (13) for perturbation quantities the following equation for u' is obtained. $\frac{\partial u^1}{\partial x} = -\frac{\partial w^1}{\partial z}$

$$U^1(t,x,z) = -\frac{Um}{l}\exp[\sigma t + i(lx+mz)] \quad ..(24)$$

In order to determine the amplitude equations for U and N eqns.(21)-(24) are substituted into (12) and (14) which result in the following equations;.from (24),

$$C_a\rho_o \frac{\partial}{\partial t}\left(\frac{\partial u^1}{\partial z} - \frac{\partial w^1}{\partial x}\right) = -\frac{\mu}{K}\left(\frac{\partial u^1}{\partial z} - \frac{\partial w^1}{\partial x}\right) + \theta\Delta\rho g \frac{\partial n^1}{\partial x} + \mu^* \frac{\partial^2 H^1}{\partial x \partial z}$$

$$C_a\rho_o \frac{\partial}{\partial t}\left(\frac{-Um}{l}\exp[\sigma t+i(lx+mz)]im - U\exp[\sigma t+i(lx+mz)]il\right)$$
$$= -\frac{\mu}{K}(\frac{-Um}{l}\exp[\sigma t+i(lx+mz)im - U\exp[\sigma t+i(lx+mz)il) + \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$
$$\theta\Delta\rho g N\exp[\sigma t+i(lx+mz)il + \mu^*\exp[\sigma t+i(lx+mz)il.im$$

Simplifying and putting . $k^2 = l^2 + m^2$, *we.have*

$$-C_a\rho_o K\sigma Uk^2 + \mu Uk^2 + \theta\Delta\rho g Nl^2K + \mu^* iQl^2Kmk^2U\left(\mu + C_a\rho_o K\sigma\right)$$
$$+\theta\Delta\rho g Nl^2K + \mu^* iQl^2Km = 0$$

Then from (25) $\quad \frac{\partial H^1}{\partial t} = \frac{\partial w^1}{\partial z} + \vartheta_m \frac{\partial^2 H^1}{\partial z^2}$

$$\frac{\partial}{\partial t}\left(Q\exp[\sigma t+i(lx+mz)]\right) = \frac{\partial}{\partial z}\left(U\exp[\sigma t+i(lx+mz)]\right) +$$
$$\vartheta_m \frac{\partial^2}{\partial z^2}\left(Q\exp[\sigma t+i(lx+mz)]\right)Q\sigma\left(U\exp[\sigma t+i(lx+mz)]\right) =$$
$$U\left(U\exp[\sigma t+i(lx+mz)]\right)im + \vartheta_m Q\left(U\exp[\sigma t+i(lx+mz)]\right)im.im$$

Simplifying we have,

$$Q = \frac{Uim}{(\sigma + \vartheta_m m^2)} \quad ..(26)$$

Then from (20)

$$\Rightarrow N\sigma + w_c Nim + w_c Bn_o U\left[(1+\alpha_o)m^2 + (1-\alpha_o)l^2\right] = -DNk^2 \quad ..(27)$$

Substituting (26) in equation (25) we have,

$$U = \frac{-gKl^2N\Delta\rho\theta(\sigma + \vartheta_m m^2).}{-\mu^* Um^2l^2K + k^2(\mu + C_a\rho_o K\sigma)(\sigma + \vartheta_m m^2)} \quad ..(28)$$

Substituting U in equation (27) we have,

$$N\sigma + w_c Nim + w_c Bn_o\left[(1+\alpha_o)m^2 + (1-\alpha_o)l^2\right]$$
$$\left(\frac{-gKl^2N\Delta\rho\theta(\sigma + \vartheta_m m^2).}{-\mu^* Um^2l^2K + k^2(\mu + C_a\rho_o K\sigma)(\sigma + \vartheta_m m^2)}\right) = DNk^2$$

where $k^2 = l^2 + m^2$. Eliminating the amplitudes from eqs.(27) and (28) results in the following dispersion equation for the growth rate parameter σ :

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$$(\sigma + imw_c + Dk^2)(-\mu^* m^2 l^2 K + k^2 \mu\sigma + k^2 C_a \rho_o K\sigma^2 + k^2 \vartheta_m m^2 \mu + k^2 C_a \rho_o K \vartheta_m m^2 \sigma) + w_c Bn_o((1+\alpha_o)m^2 + (1-\alpha_o)l^2)(-gKl^2 \Delta\rho\theta\sigma - gKl^2 \Delta\rho\theta\vartheta_m m^2) = 0$$

On simplification which gives:

$$(k^2 C_a \rho_o K)\sigma^3 + (k^2\mu + k^2 C_a \rho_o K \vartheta_m m^2 + imw_c k^2 C_a \rho_o K + Dk^4 C_a \rho_o K)\sigma^2 + (-\mu^* m^2 l^2 K + k^2 \vartheta_m m^2 \mu + imw_c k^2 \mu + im^3 w_c k^2 C_a \rho_o K \vartheta_m + Dk^4 \mu - w_c Bn_o ((1+\alpha_o)m^2 + (1-\alpha_o)l^2) gKl^2 \Delta\rho\theta + Dk^4 C_a \rho_o K \vartheta_m m^2)\sigma + (-iw_c \mu^* m^3 l^2 K + im^3 w_c k^2 \vartheta_m \mu - Dk^2 m^2 \mu^* l^2 K + Dk^4 \vartheta_m m^2 \mu - w_c Bn_o (1+\alpha_o)m^2 + (1-\alpha_o) l^2 gKl^2 \Delta\rho\theta\vartheta_m m^2) = 0 \quad \ldots\ldots(29)$$

The three roots for the growth rate parameter σ are computed from the equation (29) and a part of the root that has greater real part is presented below (since it is extremely lengthy).

$$\sigma = \frac{1}{6}(72\, k\, c_a^2\, \rho_o^2\, K^3 m^3 \mu^o l^2\, i\, W_c + 72\, c_a^2\, \rho_o^2\, K^3 m^2 \mu^o l^2 D k^3 - 36\, k\, c_a^2\, \rho_o^2\, K^3 m^4 \mu^o l^2 \nu_m - 36\, k\, c_a\, \rho_o\, K^2 m^2 \mu^o l^2 \mu - 36\, k\, c_a^2\, \rho_o^2\, K^3 W_c^2\, B\, n_o\, g\, l^2 \Delta_\rho\, \theta\, m^3\, i \quad \ldots\text{etc}\ldots\ldots\ldots\ldots\ldots\ldots(30).$$

In the above expression $\mu^o = \mu^*$. In order to prove that critical permeability exists it is absolutely necessary to prove that (i) the system is stable, when the permeability of the porous medium is close to zero and (ii) the system becomes unstable when the permeability is sufficiently large. Accordingly instability appeared only when the real part of σ is positive. The Taylor series expansion of this root about the point K=0 is found. By neglecting the quadratic and higher order terms in this expansion, the following solution (valid only for the small values of permeability) is obtained;

$$\sigma = \left(-\frac{1}{6}\frac{8^{(1/3)}(k^3\mu^3)^{(1/3)}}{k\, c_a \rho_o} - \frac{1}{12}\frac{k\mu^2 8^{(2/3)}}{c_a \rho_o (k^3\mu^3)^{(1/3)}} - \frac{1}{3}\frac{\mu}{c_a \rho_o}\right)K^{-1} + \left(\frac{1}{144} 8^{(1/3)}(k^3\mu^3)^{(1/3)}\right(12\, c_a \rho_o k^3 \nu_m m^2 \mu^2 + 12\,\mathrm{sqrt}(-3k^{10}\mu^4 D^2 - 3 i^2 m^2 W_c^2 k^6 \mu^4 - 6 D k^8 \mu^4\, i\, m\, W_c \quad \ldots\ldots\text{etc (31)}$$

From the above equation it is observed that for K=0, σ has a negative real part. $-k^2 D$ which is independent of μ^*. This suggests that for sufficiently small values of permeability the system is stable. Therefore in order to prove that the system becomes unstable with the increase K, it is absolutely necessary to show that the real part of σ will be positive. Suppose m=o (which corresponds to the case of no vertical disturbances). In this case the root of the eqn. (31) with the greater real part is found to be $\sigma =$

$$\left(-\frac{1}{6}\frac{8^{(1/3)}(k^3\mu^3)^{(1/3)}}{k\,c_a\,\rho_o}-\frac{1}{12}\frac{k\,\mu^2\,8^{(2/3)}}{c_a\,\rho_o\,(k^3\mu^3)^{(1/3)}}-\frac{1}{3}\frac{\mu}{c_a\,\rho_o}\right)K^{-1}+\left(\frac{1}{144}8^{(1/3)}(k^3\mu^3)^{(1/3)}\,(\right.$$
$$12\,c_a\,\rho_o\,k^3\,\nu_m\,m^2\,\mu^2+12\,\mathrm{sqrt}(-3\,k^{10}\,\mu^4\,D^2-3\,i^2\,m^2\,W_c^{\;2}\,k^6\,\mu^4-6\,D\,k^8\,\mu^4\,i\,m\,W_c \qquad (32)$$

The upper limit of the critical permeability is computed by solving (32) for σ =0, K_{crit}^{upper} ={}..(33)

The above equation clearly proves and predicts that the influence of magnetic field on the critical permeability is nil. In other words the above equation is exactly the same as that of the hydrodynamic case [Kuznetsov (2002)]

Results (hydromagnetic case) : The governing equations are highly complex and the analysis is extremely tedious. The most important result of this investigation is that effect of uniform magnetic field on the estimation of critical permeability is insignificant.

References

1. T.J pedley and J.O Kessler, proc.R. Soc. Lon. B231.47 (1987)
2. T.J pedley and N.A Hill and J.O Kessler, J. Fluid Mech.195.223 (1988)
3. T.J pedley and J.O Kessler, Ann. Rev. Fluid Mech.24. 313 (1992)
4. S. Gorai and N.A Hill, J. Fluid Mech. 400. 1 (1999)
5. S Whitakar, The method of volume averaging, Kluwer, Dordrecht (1999)
6. D.A. Nield and A. Bejan, Convection in porous Media. 2nd ed. Springer, New York (1999)